

# **Markscheme**

**May 2017**

**Extended mathematics**

**On-screen examination**

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The annotation **OE** meaning Or Equivalent is used in the mark scheme.

**Error Carried Forward (ECF) marks**

Errors made at any step of a solution affect all working that follows. In general, **Error Carried Forward (ECF)** marks are awarded after an error.

- a) **ECF** applies from one part of a question to a subsequent part of the question and also applies within the same part.
- b) If an answer resulting from **ECF** is inappropriate (*eg*, negative distances or  $\sin x > 1$ ) then subsequent marks should not be awarded.
- c) If a question is transformed by an error into a **simpler question** then **ECF** may not be fully awarded.
- d) To award **ECF** marks for a question part, **there must be working present for that part**.
- e) **ECF** is only applied to working which is correct. This means that all working subsequent to an error must be checked for accuracy.
- f) A misread (**MR**) is an error. **ECF** is normally awarded.

**General points**

- a) As this is an international examination, accept all alternative forms of **notation**, for example 1.9 and 1,9 or 1 000 or 1.000. However **DO NOT ACCEPT** incorrect mathematical notation e.g  $x^2$  for  $x^2$  unless noted otherwise in the markscheme.
- b) Ignore further working after a correct answer **unless** it indicates a lack of mathematical understanding **i.e. if the further working contradicts the correct answer**, then the last mark cannot be awarded.
- c) Where candidates have written two solutions to a question, mark the response that deserves more marks.
- d) In the markscheme, equivalent examples of **numerical** and **algebraic** forms or **simplified** answers will generally be written in the notes preceded by **OE** or equivalent e.g.

$$\frac{1}{2} \text{ OR } 1/2 \text{ OR } 1 \div 2 \quad \text{and} \quad \frac{x}{2} \text{ OR } x/2 \text{ OR } x \div 2$$

- e) In the markscheme, information provided in brackets indicate detail that may be seen in a candidate response but is not necessary to award the marks.
- f) Special case marks SC can be allocated instead of but not in addition to the marks prescribed in the markscheme.
- g) Accept seeing equations not in-line.
- h) Accept notation errors in intermediate steps.
- i) When a calculator screenshot is taken, accept not seeing the whole operation.

Example 1

- 1 mark awarded and corresponding notes are aligned

b	<ul style="list-style-type: none"> <li>•<sup>1</sup> Show clear line of reasoning in the method</li> <li>•<sup>2</sup> 4</li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> 45 &amp; 49 seen <b>OE</b> <i>eg</i>, <math>49 = 45 + x</math></li> <li>•<sup>2</sup> <b>ACCEPT</b> <math>4.5 + X/10 = 4.9</math> <b>and</b> Ans 4</li> </ul>	2
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Task 1

Answers			Notes	Total
1	a	$A \cap B \cap C$	1 mark ACCEPT $(A \cap B) \cap C$ ACCEPT $(A \cap B) \cap (B \cap C) \cap (C \cap A)$ or similar	1
	b	{5, 6}		1
	c	<ul style="list-style-type: none"> <li>•<sup>1</sup> For <math>A \cap (B \cup C) = \{2, 5, 6\}</math></li> <li>•<sup>2</sup> For <math>(A \cap B) \cup (A \cap C) = \{2, 5, 6\}</math></li> </ul>	Correct answers might be seen in part (d) Note: The name of the set must be seen to get the mark. for {2, 5, 6} seen without being named: Award 1 mark for {2, 5, 6} seen twice but without any name: Award 1 mark	2
	d	They are the same (or equal)	ACCEPT: It illustrates the distributive law (for sets, for $\cap$ over $\cup$ ) or $A \cap (B \cup C) = A \cap B) \cup (A \cap C)$  ACCEPT "both contain 2,5,6 WTTE,  ECF from (c) for example {2, 5, 6} and {2, 5, 7} and response in (d) - "2 and 5 are in both sets" or similar	1
	e	<ul style="list-style-type: none"> <li>•<sup>1</sup> {1, 2, 5, 6}</li> <li>Alternative method using set algebra</li> <li>•<sup>1</sup> simplifies eg <math>A \cap (B \cup B')</math> using rule from b)</li>   <li>•<sup>2</sup> (It is the same as) A.</li> </ul>	1 mark for set Do not ACCEPT partial answers for • <sup>1</sup> eg $\{2, 5, \} \cup \{1, 6\}$  • <sup>2</sup> WTTE.	2

Answers		Notes	Total	
2	a	<ul style="list-style-type: none"> <li>•<sup>1</sup> (The medians) 57 and 52 seen</li>   <li>•<sup>2</sup> This means the average age of medicine is older/bigger than physics WTTE</li>   <li>•<sup>3</sup> (The interquartile ranges) 20 and 15</li>   <li>•<sup>4</sup> This means the winners in physics are more close/less spread in age. This comment must refer to spread.</li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> Is for mathematical observation for example: (Median Q2 medicine =) 57 (Median Q2 physics =) 52</li> <li>•<sup>1</sup> for both values seen and no comment award 1 mark</li> <li>•<sup>1</sup> ACCEPT 5 seen for 1 mark</li> <li>•<sup>1</sup> and •<sup>2</sup> "in general medicine is older by 5 years" WTTE award 2 marks</li>   <li>•<sup>3</sup> Is for mathematical observation for example: (IQR for medicine 67-47=) 20 (IQR for physics 62-47 =) 15</li> <li>•<sup>3</sup> award for both values seen and no comment, accept differences 67-47; 62-47</li> <li>•<sup>3</sup> DO NOT ACCEPT 'IQR is between 47 and 67' or WTTE</li> <li>•<sup>3</sup> DO NOT ACCEPT 'the majority is between 47 and 67' or WTTE</li> <li>•<sup>3</sup> must refer to IQR not range in general</li>   <li>•<sup>4</sup> Do not accept "less skewed"</li> <li>•<sup>4</sup> ACCEPT 50% (in this range)</li> </ul>	<b>4</b>
	b	<ul style="list-style-type: none"> <li>•<sup>1</sup> 155 seen</li>   <li>•<sup>2</sup> <math>\frac{\text{their } 155}{210}</math></li> </ul>	<ul style="list-style-type: none"> <li>•<sup>2</sup> <b>OE</b> for example 0.7 <b>OR</b> 0.738 <b>OR</b> 0.74 <b>OR</b> 73.8 % <b>OR</b> 74 % <b>OR</b> better:</li> </ul>	<b>2</b>
	c	<ul style="list-style-type: none"> <li>•<sup>1</sup> 11 seen</li>   <li>•<sup>2</sup> <math>\frac{11}{\text{their } 155}</math></li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> 11 can be seen anywhere in <i>their</i> working</li>   <li>•<sup>2</sup> <b>OE</b> for example 0.071 <b>OR</b> 0.07 <b>OR</b> 7.1 % <b>OR</b> 7% <b>OR</b> better:</li> </ul>	<b>2</b>

Answers			Notes	Total
2	d	<ul style="list-style-type: none"><li>•<sup>1</sup> Applying the 20% to 210</li> <li>•<sup>2</sup> Read off the graph the corresponding age</li></ul>	<ul style="list-style-type: none"><li>•<sup>1</sup> <math>\frac{20}{100} \times 210</math> OR 42 OR <math>\frac{80}{100} \times 210</math> OR 168</li></ul>	2

Answers		Notes	Total	
3	a	<p>Step a: 14,3 . Step c: 5,7, 3</p> <ul style="list-style-type: none"> <li>•<sup>1</sup> All step a correct</li> <li>•<sup>2</sup> All step c correct</li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> 14, 3</li> <li>•<sup>2</sup> their 5, 7, their 3</li> </ul> <p>Allow ECF for from their step a</p>	2
	b	<ul style="list-style-type: none"> <li>•<sup>1</sup> Correct sum of their numbers</li> <li>•<sup>2</sup> Not divisible by 10 <i>or</i> Not a multiple of 10</li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> their 83</li> <li>•<sup>2</sup> Accept 'must be a multiple of 10'</li> </ul> <p>Sum their values correctly to a number divisible by 10 and saying it is valid because sum is divisible by 10 award •<sup>1</sup> only</p>	2
	c	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>(61 + X =) 70</math></li> <li>•<sup>2</sup> <math>(X =) 9</math></li> </ul>	<p>9 seen no working: Award 2 marks</p>	2

Answers			Notes	Total
3	d	<ul style="list-style-type: none"> <li>•<sup>1</sup> Instruction F correct</li>   <li>•<sup>2</sup> Instruction G correct</li>   <li>•<sup>3</sup> Instruction H correct</li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> (If the doubled digit is a two-digit number) add the two digits together Accept "add the two numbers together" <b>OR</b> WTTE</li>   <li>•<sup>2</sup> Add the numbers Accept "plus" the numbers <b>OR</b> "sum" the numbers <b>OR</b> WTTE</li>   <li>•<sup>3</sup> If the sum is divisible by 10 then it is valid <b>OR</b> if the sum is not divisible by 10 then not valid <b>OR</b> check if (make sure) the number is divisible by 10 <b>OR</b> WTTE The instruction has to be in the correct place to award the mark</li>   <li><b>SC:</b> Step G adding <b>and</b> if sum divisible by 10 valid. Step H adding <b>and</b> if sum not divisible by 10 invalid Award only 1 mark for •<sup>2</sup> and •<sup>3</sup> .Award •<sup>1</sup> as appropriate</li>   <li><b>SC:</b> Step G adding <b>and</b> if sum divisible by 10 valid Step H if the sum not divisible by 10 invalid Award 2 marks for •<sup>2</sup> and •<sup>3</sup> .Award •<sup>1</sup> as appropriate</li> </ul>	<b>3</b>



Answers		Notes	Total	
4	a	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>2f(x-2) = 6</math></li> <li>•<sup>2</sup> <math>f(x-2) = 3</math></li> <li>•<sup>3</sup> <math>x-2 = 6</math> <b>OR</b> <math>f(6) = 3</math></li> <li>•<sup>4</sup> <math>x = 8</math></li> </ul>	Marks awarded for <ul style="list-style-type: none"> <li>•<sup>1</sup> first algebra step, number to other side</li> <li>•<sup>2</sup> second algebra step, divide by 2</li> <li>•<sup>3</sup> refers back to table</li> <li>•<sup>4</sup> algebra step</li> </ul>	<b>4</b>
	b	<ul style="list-style-type: none"> <li>•<sup>1</sup> 0</li> <li>•<sup>2</sup> 2</li> </ul>		<b>2</b>
	c	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>f(g(0)) = f(g(4)) (= -4)</math></li>   <li>•<sup>2</sup> The inverse function of -4 has two different values</li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> May be implied</li> </ul> Accept for 2 marks <ul style="list-style-type: none"> <li>• 0 and 4 both go to -4</li> <li>• <math>f(g(x))</math> is not a one to one function</li> <li>• <math>f(g(x))</math> is a two or many – one function</li> <li>• Horizontal line test for <math>f(g(x))</math> gives two values</li> <li>• Vertical line test for the inverse of <math>f(g(x))</math> gives two values</li> <li>• The inverse function has two different values for the same value</li> <li>• for a single value of <math>x</math> there are two values of <math>y</math></li> <li>• <math>f(g(x))</math> has several values in <math>x</math> for the same value in <math>y</math>, so the reciprocal/inverse will have several values in <math>y</math> for the same value in <math>x</math></li> </ul> Accept for 1 mark The inverse function has two different values	<b>2</b>

Task 2

Answers		Notes	Total	
5	a	(BCA) = 57	1	
	b	<p><b>Alternative method 1</b></p> <ul style="list-style-type: none"> <li>•<sup>1</sup> Use of sine rule</li> <li>•<sup>2</sup> <math>\frac{100}{\sin(\text{their } 57)} = \frac{R}{\sin 3}</math></li> <li>•<sup>3</sup> Attempt to solve for R</li> <li>•<sup>4</sup> (R=) 6.240347.... (m)</li> </ul> <p>6.24 AG</p> <p><b>Alternative method 2</b></p> <ul style="list-style-type: none"> <li>•<sup>1</sup> <math>\sin 30 = \frac{BE}{100}</math> <b>OR</b> <math>\cos 30 = \frac{AE}{100}</math></li> <li>•<sup>2</sup> <math>\tan 33 = \frac{\text{their } BE + R}{\text{their } AE}</math></li> <li>•<sup>3</sup> <math>\tan 33 = \frac{\text{correct } BE + R}{\text{correct } AE}</math></li> <li>•<sup>4</sup> (R=) 6.2403478...(m)</li> </ul> <p>6.24 AG</p> <p><b>Alternative method 3</b></p> <ul style="list-style-type: none"> <li>•<sup>1</sup> <math>\cos 30 = \frac{AE}{100}</math></li> <li>•<sup>2</sup> <math>\cos 33 = \frac{\text{their } AE}{AC}</math></li> <li>•<sup>3</sup> <math>\text{their } 03.3^2 = (50\sqrt{3})^2 + EC^2</math> <b>and</b> <math>EC = 56.2403478...</math></li> <li>•<sup>4</sup> (R=) 6.2403478...(m)</li> </ul> <p>AG 6.24m</p>	<p><b>Alternative method 1</b></p> <ul style="list-style-type: none"> <li>•<sup>1</sup> 1 mark for use of sine rule</li> <li>•<sup>2</sup> 1 mark for correct substitution in sine rule</li> <li>•<sup>3</sup> 1 mark for attempt to solve (do not have to see this step)</li> <li>•<sup>4</sup> 1 mark for showing the answer before approximations</li> </ul> <p><b>Alternative method 2</b></p> <ul style="list-style-type: none"> <li>•<sup>1</sup> 1 mark for using correctly sin or cos</li> <li>•<sup>2</sup> 1 mark for using tan</li> <li>•<sup>3</sup> 1 mark for correct values used with tan</li> <li>•<sup>4</sup> 1 mark for showing the answer before approximation (may be seen as 56.24034.. earlier)</li> </ul> <p><b>Alternative method 3</b></p> <ul style="list-style-type: none"> <li>•<sup>1</sup> 1 mark for finding AE, <math>\cos 30 = AE/100</math> <b>OR</b> by inspection <math>50\sqrt{3}</math></li> <li>•<sup>2</sup> 1 mark for finding <math>AC = 103.3</math></li> <li>•<sup>3</sup> 1 mark for correct values in Pythagoras <b>and</b> find EC</li> <li>•<sup>4</sup> 1 mark for showing the answer before approximation (may be seen as 56.24034.. earlier)</li> </ul>	4

Marks		Notes	Total	
5	b	<p><b>Alternative method 4</b></p> <ul style="list-style-type: none"> <li>•<sup>3</sup> <math>\sin 33^\circ = EC/\text{their AC}</math> and their EC</li> <li>•<sup>4</sup> (R=) 6.2403478...(m)</li> </ul> <p>AG 6.24 m</p> <p><b>Alternative method 5</b> Cos rule</p> <ul style="list-style-type: none"> <li>•<sup>3</sup> <math>R = \sqrt{100^2 + 103.26^2 - (2 \times 100 \times 103.26 \times \cos 3^\circ)}</math></li> <li>•<sup>4</sup> (R=) 6.2403478...(m)</li> </ul>	<p><b>Alternative method 4</b></p> <ul style="list-style-type: none"> <li>•<sup>3</sup> 1 mark for their values in SOH and find EC</li> <li>•<sup>4</sup> 1 mark for showing the answer before approximation (may be seen as 56.24034.. earlier)</li> </ul> <p><b>Alternative method 5</b></p> <ul style="list-style-type: none"> <li>•<sup>3</sup> 1 mark for correct substitution in cosine rule</li> <li>•<sup>4</sup> 1 mark for showing the answer before approximation</li> </ul>	
	c	<ul style="list-style-type: none"> <li>•<sup>1</sup> Substituting 6.24 correctly into volume formula</li> <li>•<sup>2</sup> Volume calculated correctly using <i>their</i> radius</li> <li>•<sup>3</sup> Their value correctly approximated to nearest m<sup>3</sup></li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>\frac{4}{3}\pi(6.24)^3</math> Accept: <math>4/3 \times \pi \times 6.24^3</math></li> <li>•<sup>2</sup> <i>their</i> 1017.7529..(using <math>\pi</math>) <b>OR</b> <i>their</i> 1017.237.. (using 3.14) <b>OR</b> 323.96<math>\pi</math> <b>OE</b></li> <li>•<sup>3</sup> 1018 (m<sup>3</sup>) <b>OR</b> 1017 (m<sup>3</sup>)</li> </ul>	3

Answers			Notes	Total
5	d	<ul style="list-style-type: none"> <li>•<sup>1</sup> A valid mathematical reason</li> <li>•<sup>2</sup> Another valid reason (mathematical or not)</li> </ul>	<p>Mathematical reasons:</p> <ul style="list-style-type: none"> <li>(i) Referring to rounding of measurements <b>OR</b> calculations</li> <li>(ii) Referring to inner and outer diameter <b>OR</b> thickness of sphere</li> </ul> <p>Examples of real-life reasons:</p> <ul style="list-style-type: none"> <li>(i) Referring to shape not a perfect sphere</li> <li>(ii) Referring to the sphere is not totally filled with water (regardless of the reason)</li> <li>(iii) Expansion of water</li> </ul> <p>DO NOT ACCEPT the vertical/pipes post may include water as well            DO NOT ACCEPT referring to errors in measurements given            Do NOT ACCEPT "because maybe I made mistakes"</p>	2

5	e	<b>Marks</b>	<b>1</b>	<b>2</b>	10
		<p>(H) Calculate total water consumption for the households in the community</p>	<p>Two attempts from any of the calculations below:</p> <p>Dividing 300000 by 4 to find the number of households  <math>300\ 000/4 (= 75\ 000)</math> households  <b>OR</b>                      Multiplying their 366 by their 75000 to find the amount of water consumed per day by households                      their366 * their 75000 (= 27 450 000) litres per day  <b>OR</b>                      Dividing their 366 by 24 to find the amount of water in litres per hour for each household                      their366/24(=15.25) l/h  <b>OR</b>                      Calculating how many litres for every household during the 4 hours outage time  <b>OR</b>                      Acceptable estimation for the amount of water used by household per hour (between 1100 000 and 1200000) without calculations</p>	<p>Calculate correctly the amount of water used by households per hour:                      (27 450 000/24 =) 1 143 750 litres per hour</p> <p><b>OR</b>                      (15.25*75000=)1143750 litres per hour</p>	
		<p>(L) Calculate total water consumption for individuals in the community</p>	<p><b>One</b> attempt from the calculations below:</p> <p>Multiplying 300000 by their 51 to find the amount of water consumed per day by the individuals                      Ex: <math>300\ 000 \times \text{their}51 (= 15\ 300\ 000)</math> litres per day  <b>OR</b>                      Dividing by 24 to find amount of water consumed per individual per hour                      Their 51/24(=2.125) l/h  <b>OR</b>                      Calculating how much litres for every individual during the 4 hours outage time  <b>OR</b>                      Acceptable estimation for the amount of water used by individuals per hour (between 630000 and 650000 l/h) without calculations</p>	<p>Calculate correctly the amount of water used by individuals per hour:                      Ex: <math>(15300000/24=)637500</math> litres per hour</p> <p><b>OR</b>                      (2.125*300000=)637500 litres per hour</p>	

<b>5</b>	<b>e</b>	<b>Marks</b>	<b>1</b>	<b>2</b>
		(T) Estimate amount of time before the water runs out	Attempt to calculate estimate for time: Household $\frac{950\ 000}{\text{their } 1\ 143\ 750} (\approx 0.830\dots\text{hrs} \approx 50\ \text{mins})$ <b>OR</b> Individual $\frac{950\ 000}{\text{their } 637\ 500} (\approx 1.49\dots\text{hrs} \approx 89\ \text{mins})$ <b>OR</b> attempt for household and individuals but wrong result  $\frac{950\ 000}{\text{their } 1\ 143\ 750 + \text{their } 637\ 500} (\approx \text{not } 0.5)$	Calculating their estimate combining both household and individual information correctly  $\left(\frac{950\ 000}{\text{their } 1\ 143\ 750 + \text{their } 637\ 500}\right) \approx 0.5(33\dots)\text{hrs} \approx 30\ \text{to } 35\ \text{mins}$
		(A) Advice for the community	Advice related to minimizing the use in general <b>OR</b> Advise related to minimize/stop one specific activity	Advice requesting clearly to stop/reduce at least two of the following activities that consume too much water: Showers, wash dishes, washing clothes, or garden watering
(J) Justification of degree if accuracy	Weak justification not supported  Examples: This is just an estimate <b>OR</b> We never know for sure the actual amount <b>OR</b> This is just an average <b>OR</b> Correct and sensible rounding for their value(s) of time Example 30 min or 0.5 hours  <b>Do not accept:</b> "my time is very accurate because I made the correct calculations"	Good justification supported  Examples: Comment that the time calculated is if all population do not respond to advice and that the estimate of time can be more if they respond to advice <b>OR</b> Calculate time if they respond to advice <b>OR</b> the number of persons per household is an average so not accurate <b>OR</b> Referring to fact that during the specific 4 hours of outage not all activities considered in calculations are actually performed		
<p><b>ATTENTION:</b>                      Seeing the total consumption for households and individuals (1 143 750 + 637500 =) 1781250 l/h allows the award of H2 and L2</p>				

Notes Ignore incorrect units Apply ECF in E and J		Total 10 marks	
6	Marks	1	2
C	Calculates the area of the Circle	$\pi \times 10^2$ method/formula eg finds area of circle. Seen or implied at T	
T	Calculation for the equilateral Triangle	$\frac{1}{6}$ of circle seen <b>OR</b> $3 \times \frac{1}{6}$ of circle seen <b>OR</b> $\frac{1}{2}$ of circle seen	Their area of three sectors ( $50\pi$ ) <b>OR</b> their157(.0796327)
A	Correct Area for the triangle without a signal	$173 - 50\pi$ , accept their $50\pi$ , sets up subtraction <b>OR</b> 15.9(...) <b>OR</b> 16 <b>OR</b> answer to $173 - \text{their } 157(.0796327)$	
S	Calculation for the Square without a signal	Implies that full circle inside the square <b>OR</b> $4 \times \frac{1}{4}$ of circle seen <b>OR</b> $100\pi$	$400 - \pi \times 10^2$ sets up subtraction <b>OR</b> 85.8(...) <b>OR</b> 86
E	Comparison of the Efficiency of the two ways the transmitters have been positioned	Triangle is smaller in area than the square <b>OR</b> Triangle has a smaller area without signal than the square (Their 85.8 – their 15.9)	Triangle is smaller in area than the square <b>AND</b> Triangle has a smaller area without signal than the square (Their 85.8 – their 15.9)
J	Justification of the most efficient layout	One numerical statement/comparison from eg <ul style="list-style-type: none"> <li>• Triangle has 9.2% not covered</li> <li>• Square has 21.45% not covered</li> <li>• Double the triangle to 346 and its 31.86 not covered</li> <li>• Doubling the triangle shows that there is a lot smaller area not covered than the square</li> </ul>	Two numerical statement/comparison from eg <ul style="list-style-type: none"> <li>• Triangle has 9.2% not covered</li> <li>• Square has 21.45% not covered</li> <li>• Double the triangle to 346 and its 31.86 not covered</li> <li>• Doubling the triangle shows that there is a lot smaller area not covered than the square</li> </ul>

Task 3

Answers			Notes						Total																		
7	a	<ul style="list-style-type: none"> <li>•<sup>1</sup> Any three correct: award 1 mark</li> <li>•<sup>2</sup> All correct</li> </ul>	<table border="1"> <tr> <td>3</td> <td><math>\frac{1}{6}</math></td> <td><math>\frac{1}{12}</math></td> <td><math>\frac{-1}{6}</math></td> <td><math>\frac{1}{12}</math></td> <td><math>\frac{-1}{12}</math></td> </tr> <tr> <td>4</td> <td><math>\frac{1}{8}</math></td> <td><math>\frac{1}{16}</math></td> <td><math>\frac{-1}{8}</math></td> <td><math>\frac{1}{16}</math></td> <td><math>\frac{-1}{16}</math></td> </tr> <tr> <td>5</td> <td><math>\frac{1}{10}</math></td> <td><math>\frac{1}{20}</math></td> <td><math>\frac{-1}{10}</math></td> <td><math>\frac{1}{20}</math></td> <td><math>\frac{-1}{20}</math></td> </tr> </table>						3	$\frac{1}{6}$	$\frac{1}{12}$	$\frac{-1}{6}$	$\frac{1}{12}$	$\frac{-1}{12}$	4	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{-1}{8}$	$\frac{1}{16}$	$\frac{-1}{16}$	5	$\frac{1}{10}$	$\frac{1}{20}$	$\frac{-1}{10}$	$\frac{1}{20}$	$\frac{-1}{20}$	2
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b	$y_C = -\frac{1}{4k}$							1																			
c	<ul style="list-style-type: none"> <li>•<sup>1</sup> Completing the pattern for another value not in the table <math>k \geq 6</math> adding 4 to denominator (20+4=24 seen)</li> <li>•<sup>2</sup> Calculating <math>y_C = -\frac{1}{4(6)} = -\frac{1}{24}</math></li> <li>•<sup>3</sup> Comment that they are equal</li> </ul>	Award 1 mark for testing a value from the table $k \leq 5$ but only if the three verify steps are seen <ul style="list-style-type: none"> <li>• value from the table</li> <li>• calculate from the rule</li> <li>• say they are the same/values are correct.</li> </ul>						3																			
d	<p><b>Alternative method 1</b></p> <ul style="list-style-type: none"> <li>•<sup>1</sup> <math>\frac{y_A - y_C}{x_A - x_C} = 1</math> and <math>\frac{y_A - y_C}{\frac{1}{2k} - 0} = 1</math></li> <li>•<sup>2</sup> <math>\frac{k(x_A)^2 - y_C}{\frac{1}{2k} - 0} = 1</math> OR <math>\frac{k\left(\frac{1}{2k}\right)^2 - y_C}{\frac{1}{2k} - 0} = 1</math></li> <li>•<sup>3</sup> <math>k(x_A)^2 - y_C = \frac{1}{2k}</math> OR <math>k\left(\frac{1}{2k}\right)^2 - y_C = \frac{1}{2k}</math></li> </ul> <p><math>y_C = -\frac{1}{4k}</math> AG</p>	<ul style="list-style-type: none"> <li>•<sup>1</sup> Gradient formula = 1 and correct substitution for <math>x_A</math></li> <li>•<sup>2</sup> Correctly substitutes <math>y_A = k(x_A)^2</math> or <math>y_A = k\left(\frac{1}{2k}\right)^2</math></li> <li>•<sup>3</sup> further working to <math>y_C = -\frac{1}{4k}</math></li> </ul> <p><math>y_C = -\frac{1}{4k}</math> is seen at part b) and here it is the same as AG</p>						3																			



		Answers	Notes	Total
7	d	<p><b>Alternative method 2</b></p> <p>•<sup>1</sup> <math>\frac{k(x_A)^2 - y_C}{\frac{1}{2k} - 0} = 1</math></p> <p>•<sup>2</sup> <math>k(x_A)^2 - y_C = \frac{1}{2k}</math></p> <p>•<sup>3</sup> <math>k\left(\frac{1}{2k}\right)^2 - y_C = \frac{1}{2k}</math></p> <p><math>-y_C = \frac{1}{2k} - k\left(\frac{1}{2k}\right)^2</math></p> <p><math>-\frac{1}{4k} = y_C</math></p> <p>AG</p>	<p>•<sup>1</sup> Gradient formula = 1 and correct substitution for <math>x_A</math> and <math>y_A = k(x_A)^2</math></p> <p>•<sup>2</sup> Cross multiplies correctly</p> <p>•<sup>3</sup> Substitutes <math>k\left(\frac{1}{2k}\right)^2</math> for <math>k(x_A)^2</math> and further working to <math>y_C = -\frac{1}{4k}</math></p> <p><math>y_C = -\frac{1}{4k}</math> is seen at part b) and here it is the same as AG</p> <p><b>SC</b></p> <p>Award 2 mark for a "proof" similar to example below. It is based on <math>y_A = -y_C</math> which has been found by inspection.</p> $\frac{y_A - y_C}{x_A - x_C} = 1$ $\frac{-y_C - y_C}{\frac{1}{2k} - 0} = 1$ $-2y_C = \frac{1}{2k}$ $-\frac{1}{4k} = y_C$	

Answers			Notes	Total
7	e	$X_c$ has the same value as $p$	$X_c = p$	1
	f	<p><b>Alternative method 1</b></p> <ul style="list-style-type: none"> <li>•<sup>1</sup> The denominators of <math>Y_c</math> are always 8</li> <li>•<sup>2</sup> The numerator of <math>Y_c</math> goes up by 8</li> </ul> <p><b>Alternative method 2</b></p> <ul style="list-style-type: none"> <li>•<sup>1</sup> It is an arithmetic progression</li> <li>•<sup>2</sup> The common difference is 1</li> </ul>	<p>ACCEPT:</p> <p>The values of <math>Y_c</math> go up by 1 : award 2 marks</p>	2
	g	<ul style="list-style-type: none"> <li>•<sup>1</sup> The rule can be obtained by substituting <math>k = 2</math> into <math>-1/4k</math> and then testing/trial and error</li> </ul> <p><b>OR</b></p> <p>by inspection 0 -1/8; 1 - 1/8 etc</p> <ul style="list-style-type: none"> <li>•<sup>2</sup> <math>y_c = \frac{8q-1}{8}</math>    <b>OR</b>    <math>y_c = q - \frac{1}{8}</math></li> </ul> <p><b>OR</b></p> <p>If no working seen award 1 for each part</p>	<ul style="list-style-type: none"> <li>•<sup>1</sup> for suitable working</li> <li>•<sup>2</sup> working can be implied by a correct answer.</li> </ul>	2

7	h	<table border="1"> <thead> <tr> <th>Mark</th> <th>Predictions P</th> <th>Description D</th> <th>Testing T</th> <th>Verifying V</th> <th>Justify/proof J</th> <th>Notation and terminology N</th> <th>Communication C</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>Attempted to make predictions</td> <td>Attempted to describe a pattern</td> <td>Attempted to test their described pattern <b>OR</b> general rule</td> <td>Attempted to verify their described pattern or general rule</td> <td>Attempted to justify their described pattern <b>OR</b> general rule</td> <td>The notation <b>OR</b> terminology are not always correct</td> <td>No communication. Only calculations or algebraic steps</td> </tr> <tr> <td>2</td> <td>Correctly predicted one term</td> <td>Correctly described one pattern <b>OR</b> a simple pattern</td> <td>Tested correctly their described pattern</td> <td>Verified correctly their described pattern <b>OR</b> general rule</td> <td>Justified their general rule correctly</td> <td>The notation and terminology are correct  <b>Award only if D3 is awarded</b></td> <td>Some coherent communication</td> </tr> <tr> <td>3</td> <td>Correctly predicted more than one term Accept predictions with <math>p = 0</math> <b>OR</b> <math>q = 0</math></td> <td>Correctly described more than one pattern <b>OR</b> one complex pattern</td> <td>Uses a correct general rule to generate a value given in the table  <b>Award only if D5 is awarded</b></td> <td></td> <td>Attempted to prove their general rule</td> <td></td> <td>Good coherent communication  <b>Award only if J2 is achieved</b></td> </tr> <tr> <td>4</td> <td></td> <td>Attempted to describe pattern as general rule</td> <td></td> <td></td> <td>Correctly proved their general rule</td> <td></td> <td></td> </tr> <tr> <td>5</td> <td></td> <td>Correctly described pattern as general rule</td> <td></td> <td></td> <td></td> <td></td> <td></td> </tr> </tbody> </table>	Mark	Predictions P	Description D	Testing T	Verifying V	Justify/proof J	Notation and terminology N	Communication C	1	Attempted to make predictions	Attempted to describe a pattern	Attempted to test their described pattern <b>OR</b> general rule	Attempted to verify their described pattern or general rule	Attempted to justify their described pattern <b>OR</b> general rule	The notation <b>OR</b> terminology are not always correct	No communication. Only calculations or algebraic steps	2	Correctly predicted one term	Correctly described one pattern <b>OR</b> a simple pattern	Tested correctly their described pattern	Verified correctly their described pattern <b>OR</b> general rule	Justified their general rule correctly	The notation and terminology are correct  <b>Award only if D3 is awarded</b>	Some coherent communication	3	Correctly predicted more than one term Accept predictions with $p = 0$ <b>OR</b> $q = 0$	Correctly described more than one pattern <b>OR</b> one complex pattern	Uses a correct general rule to generate a value given in the table  <b>Award only if D5 is awarded</b>		Attempted to prove their general rule		Good coherent communication  <b>Award only if J2 is achieved</b>	4		Attempted to describe pattern as general rule			Correctly proved their general rule			5		Correctly described pattern as general rule						22
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**Exemplification**

**-Prediction:** Makes entries in the table. Accept predictions with  $p = 0$  or  $q = 0$

**-Description of the pattern:**  $x_c = p$   
 $y_c$  denominator multiple of 4 ( $4k$ ), numerator  $4kq - 1$   $y_c \uparrow q$

**-General rule :**  $x_c = p$  and  $y_c = -1/4k + q$  OR  $y_c = \frac{-1 + 4kq}{4k}$  OR  $y_c = q - \frac{1}{4k}$

**- Testing** the rule using values given in the table

Ex.  $k=2, p=2, q=1$   $x_c = 2$   $y_c = 7/8 \Rightarrow x_c = p = 2$  and this agrees with table  $y_c = q - \frac{1}{4k}$   $y_c = 1 - \frac{1}{4(2)} = \frac{7}{8}$  and same as the table

**- Verifying** the rule by one value not given in the table

Ex.  $k = 4, p = 2, q = 1, x_c = 2$   $y_c = 15/16 \Rightarrow x_c = p = 2$  and this agrees my table  $y_c = q - \frac{1}{4k}$   $y_c = 1 - \frac{1}{4(4)} = \frac{15}{16}$  and this agrees with the table

**- Proving**

$C(0, y_c)$  where  $y_c = -1/4k$  so  $C(0, -1/4k)$

After  $(p, q)$  is applied  $\Rightarrow C$  become  $C(0 + p, -1/4k + q)$

So  $x_c = p$  and  $y_c = -1/k + q$  OR  $y_c = \frac{-1 + 4kq}{4k}$